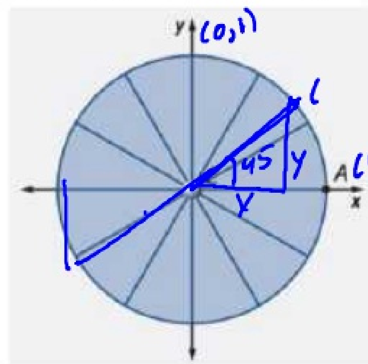
What you will learn about:
Modeling Circular Motion

The Ferris wheel was invented in 1893 as an attraction at the World Columbian Exhibition in Chicago, and it remains a popular ride at carnivals and amusement parks around the world.

COORDINATE POINTS ON A ROTATING WHEEL

Imagine that a small Ferris wheel has a radius of 1 decameter (about 33 feet) and that your seat is at a point A when the wheel begins to turn counterclockwise about its center C.



(x,y)

a. How does the x-coordinate of your seat change as the wheel turns?
Start at maximum x-value
x-values Decrease then turn negative and continue to decrease to our minimum
Start to increase again, turn positive and continue to increase back to maximum.

b. How does the y-coordinate of your seat change as the wheel turns? Start at zero increase to max then Decrease back to zero, turns negative and decrease to minimum, increase back to zero.

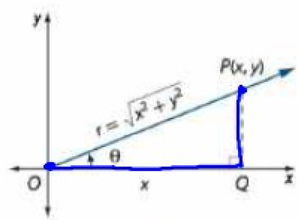
Find the angle of rotation between 0° and 360° that will take the seat from point A to the following points.

a. Maximum and minimum distance from the horizontal axis. max → 90°
min → 270°

(-,+) (+,+)

(-,-) (+,-)

SOH-CAH-TOA



$\cos\theta = \frac{Adj}{Hy} = \frac{x}{r}$

$\sin\theta = \frac{Opp}{Hyp} = \frac{y}{r}$

$\tan\theta = \frac{opp}{Adj} = \frac{y}{x}$

r

b. Maximum and minimum distance from the vertical axis. max → 0°

min → 180°

c. Points with equal x-and y-coordinates

45°, 225°

d. Points with opposite x- and y-coordinates

135°, 315°

When a circle like that modeling the Ferris wheel is placed on a rectangular coordinate grid with center at the origin (0, 0), you can use what you know about geometry and trigonometry to find the x- and y-coordinates of any point on the circle.

Find the coordinates of points that tell the location of the Ferris wheel seat that begins at point A(1, 0) when the wheel undergoes the following rotations. Record the results on a sketch that shows a circle and the points with their coordinates labels.

$\theta = 30^\circ$ $\cos 30^\circ = \frac{x}{1}$ $x = .8660$ $(.8660, .5)$ $\sin 30^\circ = \frac{y}{1}$ $y = .5$

$\theta = 70^\circ$ $x = \cos 70^\circ = .34$ $y = \sin 70^\circ = .93$

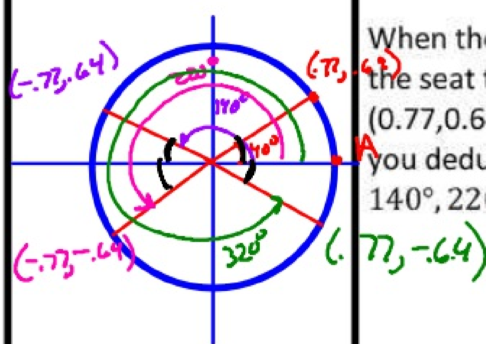
$\theta = 90^\circ$ $(0,1)$

$\theta = 120^\circ$ $(-.5, .8660)$

$\theta = 140^\circ$ $(-.76, .64)$

$\theta = 180^\circ$ $(-1, 0)$

$\theta = 220°$	$\theta = 270°$	$\theta = 310°$
$(-.76, -.64)$	$(0, -1)$	$(.64, -.76)$



When the Ferris wheel has rotated through an angle of 40°, the seat that started at A(1, 0) will be at about A' (0.77,0.64). Explain how the symmetry of the circle allows you deduce the location of the seat after rotations of $140°, 220°,\ 320°$ and some other angles as well.

Suppose that P(x, y) is a point on the Ferris wheel model with $m\angle PCA = \theta$ in degrees.

a. What are the coordinates of x and y?

$x = \cos\theta$

$y = \sin\theta$

b. How will the coordinates values be different if the radius of the circle is r decameters?

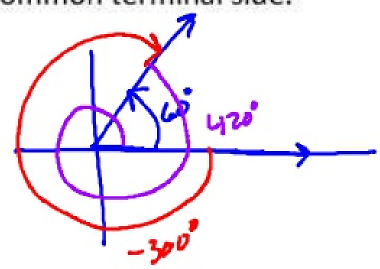
$\cos\theta = \frac{x}{r}$ $\qquad$ $\sin\theta = \frac{y}{r}$

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

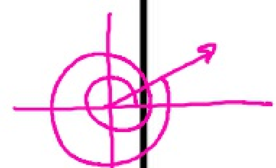
With your calculator set in degree mode, graph the functions $\cos\theta\ and\ \sin\theta\ for\ 0° \le \theta \le 360°$. Compare the patterns in those of graphs to your ideas in first problem.

How will the x- and y- coordinates of you seat change during a second complete revolution? How will those patterns be represented in graphs of the coordinate functions for $360° \le \theta \le 720°$?

Coterminal Angles – Angels in standard position (angles with the initial side on the positive x-axis) that have a common terminal side.



Find the coterminal angle between $0°$ *and* $360°$.

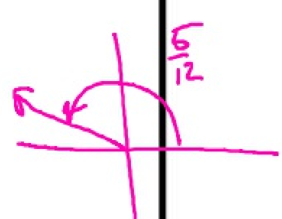


-330°	-450°	750°
$-330° + 360$	$-450 + 360$	$750 - 360$
$30°$	$-90 + 360$	$390 - 360$
	$270°$	$30°$

640°	-275°	-1034°
$640 - 360$	$-275 + 360$	$-1034 + 360$
$280°$	$85°$	$-674 + 360$
		$-314 + 360$
		46

Converting degree measure to radian measure.

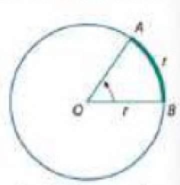
Degree measure $\left(\frac{\pi}{180}\right)$



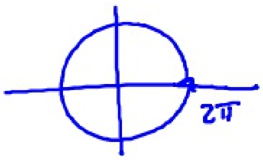
Finding revolutions equivalent to the angle.

$$\frac{\text{angle measure (Degrees)}}{360°}$$

Radian Measure = A radian is the measure of any central angle in a circle that intercepts an arc equal in length to the radius of the circle.



length $\widehat{AB} = r =$ radius
$m\angle AOB = 1$ radian



$360^\circ = 2\pi$

Find the measures in radians and revolutions equivalent to these degree measures.

90°	150°	75°	210°
$90\left(\frac{\pi}{180}\right)$	$150\left(\frac{\pi}{180}\right)$	$\frac{5\pi}{12}$	$\frac{7\pi}{6}$
$\frac{\pi}{2}$	$\frac{5\pi}{6}$		
$\frac{90}{360} = \frac{1}{4}$	$\frac{150}{360} = \frac{5}{12}$	$\frac{5}{24}$	$\frac{7}{12}$

-36°	-135°
$-\frac{\pi}{5}$	$-\frac{3\pi}{4}$
$-\frac{1}{10}$	$-\frac{3}{8}$

Find the coterminal angle between $0\ and\ 2\pi$.

$\frac{11\pi}{3} - \frac{2\pi}{1}$	$-\frac{35\pi}{18} + 2\pi$	$\frac{15\pi}{4} - \frac{8\pi}{4}$	$-\frac{19\pi}{12} + \frac{24\pi}{12}$
$\frac{11\pi}{3} - \frac{6\pi}{3}$	$-\frac{35\pi}{18} + \frac{36\pi}{18}$	$\frac{7\pi}{4}$	$\frac{5\pi}{12}$
$\frac{5\pi}{3}$	$\frac{\pi}{18}$		

Convert from radian measure to degree measure.

Find the measures in degree and revolutions equivalent to these radian measures.

$\frac{\pi}{3}$ $\qquad \frac{5\pi}{4}$ $\qquad \frac{2\pi}{5}$ $\qquad -\frac{15\pi}{16}$